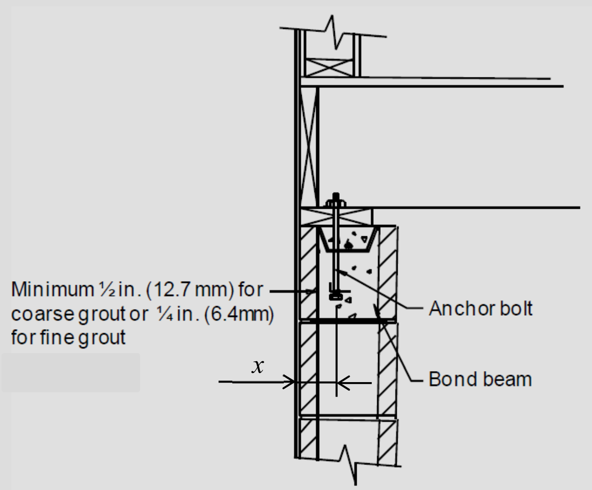
Example 9.3-6: Strength Design of a Headed Anchor Bolt Loaded in Tension Near the Edge of a Wall

Using strength design, compute the design tensile strength of a 1/2-in. diameter, A307 headed anchor bolt, embedded vertically near the edge of a bond beam in the top of a nominal 8-in. wall with a specified compressive strength, f'_m, of 2,000 psi. A section view is shown to the right, which is TMS 402 Figure CC-6.3-2. Assume the bottom of the anchor bolt is embedded a distance of 5.0 in. The effective embedment, l_b, is 5.0 in. minus the head thickness of 11/32 in., or 4.66 in. (TMS 402 Section 6.3.4).



Define x as the distance from the face of the masonry wall to the centerline of the bolt. The minimum value of x for a ½ in. bolt is:

$$x_{min} = t_{fs} + grout\ cover + C/2 = 1.25\text{ in.} + 0.5\text{ in.} + 1.01\text{ in.}/2 = 2.26\text{ in.}$$

where t_{fs} is the thickness of the face shell, ½ in. is the required clearance between the anchor bolt and the masonry for coarse grout (TMS 402 Section 6.3.1), and C is the width across the corners (MDG Table 9.3.1). Although most masonry anchor bolts have a standard hex head, a heavy hex head is conservatively assumed. Use 2-1/2 inch from the outside face of the masonry to the centerline of the bolt.

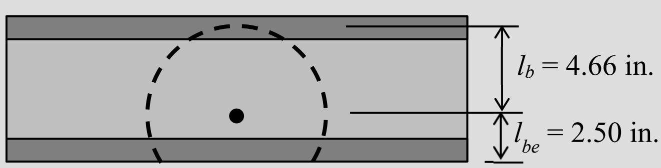
Projected tensile area:

The projected area is shown in the following figure. This is like the first row of MDG Table 9.3.5.

$$\theta = 2\arccos\left(\frac{l_{be}}{l_b}\right) = 2\arccos\left(\frac{2.5\text{ in.}}{4.66\text{ in.}}\right) = 115.1°$$

$$A_{pt} = \pi l_b^2 - \frac{l_b^2}{2}\left(\frac{\pi\theta}{180} - \sin\theta\right) = \pi(4.66\text{ in.})^2 - \frac{(4.66\text{ in.})^2}{2}\left(\frac{\pi(115.1°)}{180} - \sin(115.1°)\right) = 56.2\text{ in.}^2$$

This is 90.5% of the projected tension area when the bolt was centered, Example 9.3-4, or there is about a 10% reduction in the projected tension area, and thus the breakout strength.



Plan View

Tensile capacity:

From Example 9.3-4, masonry breakout controls.

Masonry breakout: $B_{anb} = 4.0 A_{pt}\sqrt{f'_m} = 4.0(56.2\text{ in.}^2)\sqrt{2{,}000\text{ psi}} = 10{,}050\text{ lb}$

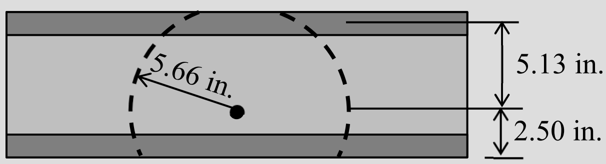
The design tensile strength is $\phi B_{anb} = 0.5(10{,}050\text{ lb}) = 5{,}030\text{ lb}$

Example 9.3-6a: Strength Design of a Headed Anchor Bolt Loaded in Tension Near the Edge of a Wall

Rework Example 9.3-6 with an effective embedment length of 5.66 in. instead of 4.66 in.

Projected tensile area:

The projected area is shown in the following figure. In this case the conical tension area falls outside both sides of the wall. The projected tension area can be determined using the last row of Table 9.3.5 twice. In the first case, solve for the projected tension area using the specified thickness, t_{sp}, equal to twice the closest distance to a face. In the second case, solve for the projected tension area using the specified thickness, t_{sp}, equal to twice the furthest distance from the face. The actual projected tension area will be the average of the two.



Plan View

The closest distance to a face is 2.5 in. Set $t_{sp} = 2(2.5\text{ in.}) = 5.0\text{ in.}$

$$X = \sqrt{l_b^2 - (t_{sp}/2)^2} = \sqrt{(5.66\text{ in.}) - (5.0\text{ in.}/2)^2} = 5.08\text{ in.}$$

$$\theta = 2\arcsin\left(\frac{t_{sp}/2}{l_b}\right) = 2\arcsin\left(\frac{5.0\text{ in.}/2}{5.66\text{ in.}}\right) = 52.4°$$

$$A_{pt} = 2Xt_{sp} + l_b^2\left(\frac{\pi\theta}{180} - \sin\theta\right)$$

$$= 2(5.08\text{ in.})(5.0\text{ in.}) + (5.66\text{ in.})^2\left(\frac{\pi(52.4°)}{180} - \sin(52.4°)\right) = 54.7\text{ in.}^2$$

The furthest distance to a face is 7.63 in. – 2.5 in. = 5.13 in. Set $t_{sp} = 2(5.13\text{ in.}) = 10.25\text{ in.}$

$$X = \sqrt{l_b^2 - (t_{sp}/2)^2} = \sqrt{(5.66\text{ in.})^2 - (10.25\text{ in.}/2)^2} = 2.40\text{ in.}$$

$$\theta = 2\arcsin\left(\frac{t_{sp}/2}{l_b}\right) = 2\arcsin\left(\frac{10.25\text{ in.}/2}{5.66\text{ in.}}\right) = 129.8°$$

$$A_{pt} = 2Xt_{sp} + l_b^2\left(\frac{\pi\theta}{180} - \sin\theta\right)$$

$$= 2(2.40\text{ in.})(10.25\text{ in.}) + (5.66\text{ in.})^2\left(\frac{\pi(129.8°)}{180} - \sin(129.8°)\right) = 97.2\text{ in.}^2$$

The projected tension area would be the average of the two values, or

$$A_{pt} = \frac{1}{2}\left(54.7 \text{ in.}^2 + 97.2 \text{ in.}^2\right) = 75.9 \text{ in.}^2$$

Tensile capacity:

Masonry breakout: $B_{anb} = 4.0 A_{pt}\sqrt{f'_m} = 4.0\left(75.9 \text{ in.}^2\right)\sqrt{2{,}000 \text{ psi}} = 13{,}580 \text{ lb}$

The design tensile strength is $\phi B_{anb} = 0.5\left(13{,}580 \text{ lb}\right) = 6{,}790 \text{ lb}$

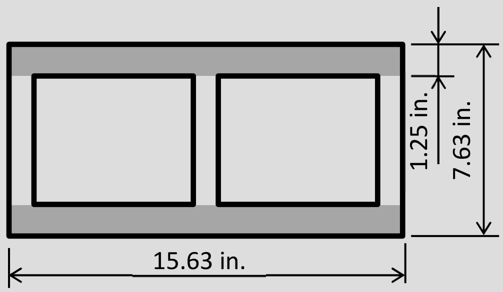
This is greater than the anchor bolt steel design strength of 6,390 lb (see Example 9.3-2), and thus anchor bolt steel strength controls. A slightly shorter bolt could be used, but since the masonry breakout design strength is only 6% greater than the steel design strength, this is an efficient design.

Example 11.3-1: Design of an Unreinforced Interior Partition Wall

Check the design of a 12 ft high vertically spanning unreinforced, ungrouted masonry partition wall constructed of 8 in. light-weight hollow CMU units, Type N masonry cement mortar, and face shell bedding. Per ASCE/SEI 7-22 Section 4.3.4, partition walls need to be designed for a minimum of a 5 psf out-of-plane load. The wall is in a low seismic area so the 5 psf governs.

Cross-sectional properties:

The critical wall section occurs in a bed joint, where there is only mortar on the face shells, as is illustrated below. The geometric properties of the wall can be obtained from MDG Appendix A, Tables A.2-2, A.2-3, and A.2-4. The calculations are illustrated for a 1 ft section.



$$A_n = 2(1.25 \text{ in.})(12 \text{ in./ft}) = 30 \text{ in.}^2/\text{ft}$$

$$I_n = \frac{1}{12}(12 \text{ in./ft})\left[(7.63 \text{ in.})^3 - (7.63 \text{ in.} - 2(1.25 \text{ in.}))^3\right] = 308.7 \text{ in.}^4/\text{ft}$$

$$S_n = \frac{I_n}{c} = \frac{308.7 \text{ in.}^4/\text{ft}}{\frac{7.63 \text{ in.}}{2}} = 81.0 \text{ in.}^3/\text{ft}$$

Material properties:

Specified compressive strength, f'_m: From Table 2 of TMS 602 and a unit strength of 2,000 psi (ASTM C90 minimum strength), $f'_m = 2{,}000$ psi.

Allowable stresses:

Flexural tension: From TMS 402 Table 8.2.4.2 (MDG Table 11.3.2), stress normal to the bed joints, hollow units, ungrouted, Type N masonry cement mortar, the allowable flexural tension stress = 12 psi.

Loads:

Out-of-plane load: 5 psf

Wall weight: 31 psf (ASCE/SEI 7-22 Table C3.1-1a, light-weight units, no grout). With unreinforced masonry, the wall weight will be significant in terms of resisting loads and is included in the design.

Check flexural tension stress:

The critical section will be the midheight of the wall. Check only the flexural tension stress. With no applied axial load other than the wall weight, the flexural compression stress will not control. The out-of-plane load is defined as a live load by ASCE/SEI 7. There is no load combination that addresses when the primary load is a live load and the dead load is acting as a "resistance". It is reasonable to use 0.6*D* for flexural tension to be consistent with other load combinations for allowable stress design where the dead load is acting as a "resistance".

$$-\frac{P}{A_n}+\frac{M}{S_n}=-\frac{0.6(31\text{ psf})(12\text{ ft}/2)}{30\text{ in.}^2/\text{ft}}+\frac{\left[5\text{ psf}(12\text{ ft})^2/8\right](12\text{ in./ft})}{81.0\text{ in.}^3/\text{ft}}=-3.7+13.3=9.6\text{ psi}$$

Since the flexural tension stress is less than the allowable flexural tension stress (9.6 psi < 12 psi), the design is adequate. Note that if the axial stress had been ignored the design would not be adequate.

Check shear stress:

The allowable unreinforced shear stress equations were developed for in-plane shear and are not directly applicable to out-of-plane shear. There are two shear limit states: diagonal cracking and sliding.

TMS 402 8.2.6.2 (a) is for the limit state of diagonal cracking. The allowable shear stress is:

$$F_v=1.5\sqrt{f'_m}=1.5\sqrt{2{,}000\text{ psi}}=67.1\text{ psi}$$

A reasonable approach is to compare this allowable stress to the stress in the webs of the units. Specify a unit with three webs, and assume the web width is the ASTM C90 minimum web width of 3/4 in. The shear area per ft of wall would be:

$$A_n=\frac{3(7.63\text{ in.})(0.75\text{ in.})}{16\text{ in.}}\frac{12\text{ in.}}{\text{ft}}=12.9\frac{\text{in.}^2}{\text{ft}}$$

Shear force: $V=\dfrac{wh}{2}=\dfrac{5\text{ psf}(12\text{ ft})}{2}=30\dfrac{\text{lb}}{\text{ft}}$

Applied shear stress (MDG Eq. 11.3-1): $f_v=\dfrac{3}{2}\dfrac{V}{A_n}=\dfrac{3}{2}\dfrac{30\text{ lb/ft}}{12.9\text{ in.}^2/\text{ft}}=3.5\text{ psi}$

The applied shear stress of 3.5 psi is much less than the allowable shear stress of 67.1 psi.

TMS 402 Table 8.2.6.2 is for the limit state of sliding. Use the minimum allowable shear stress of 15 psi. If sliding were to occur, it would be at the bed joint. Use the net area of the bed joint.

Applied shear stress (MDG Eq. 11.3-1): $f_v=\dfrac{3}{2}\dfrac{V}{A_n}=\dfrac{3}{2}\dfrac{30\text{ lb/ft}}{30.0\text{ in.}^2/\text{ft}}=1.5\text{ psi}$

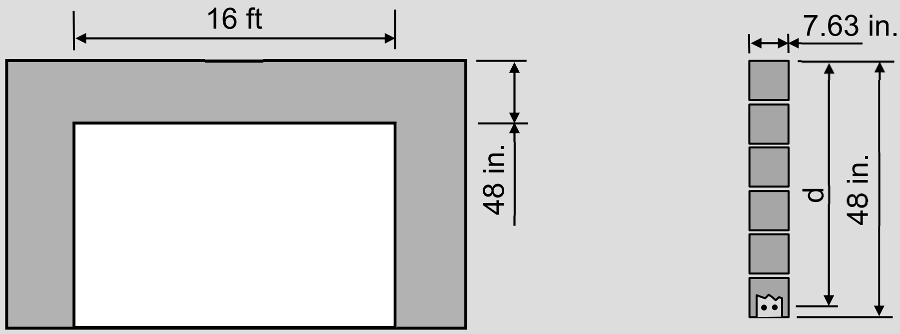
The applied shear stress of 1.5 psi is much less than the allowable shear stress of 15 psi.

For both diagonal cracking and sliding, the applied shear stress is much less than the allowable shear stress.

MDG Tip: For unreinforced walls loaded out-of-plane, the allowable shear stress will not govern and need not be checked.

Example 12.4-2A: Lintel Design according to Strength Provisions (Concrete Masonry)

A uniformly distributed load of 700 lb/ft dead load and 300 lb/ft roof live load is applied at the roof level of the structure shown below. Grade 60 steel, Type S Portland cement/lime mortar, and 8 in. medium weight CMU are used. The lintel has a clear span of 16 ft, and a total nominal depth (height of parapet plus distance between the roof and the lintel) of 4 ft. Design the lintel.



ELEVATION

CROSS-SECTION

Material properties:

f'_m = 2,000 psi (TMS 402 Table 2, unit strength of 2,000 psi)

$E_m = 900f'_m$ = 1,800,000 psi (TMS 402 Table 4.2.2)

Span length:

The opening may have a movement joint placed on either or both sides, at a distance of one-half the unit length from the opening. The lintel therefore bears on two bearing areas, each of length 8 in. TMS 402, Section 5.3.1.1 defines span length of beams as the distance from face-to-face of supports, plus ½ of the required bearing length at each end. The span is 16 ft plus ½(8 in.) plus ½(8 in.), or 16.67 ft.

Required depth of lintel:

Determine the required depth of the lintel to avoid the use of shear reinforcement. Initially assume the entire depth is fully grouted and a weight of 81 psf (ASCE/SEI 7, medium weight units). TMS 402 Section 5.3.1.5 permits the lintel to be designed for shear at $d/2$ from the face of supports. Initially design for the shear at the reaction, using the load combination $1.2D + 1.6L_r$.

$$w_u = 1.2\left(700 \text{ lb/ft} + 4 \text{ ft}\left(81 \text{ psf}\right)\right) + 1.6\left(300 \text{ lb/ft}\right) = 1{,}710 \text{ lb/ft}$$

$$V_u = \frac{w_u l}{2} = \frac{\left(1{,}710 \text{ lb/ft}\right)16.67 \text{ ft}}{2} = 14{,}250 \text{ lb}$$

The minimum required depth to avoid shear reinforcement can be determined from MDG Eq. 12.4-8.

$$d_{min} = \frac{V_u}{1.8 t_{sp}\sqrt{f'_m}} = \frac{14{,}250 \text{ lb}}{1.8\left(7.63 \text{ in.}\right)\sqrt{2{,}000 \text{ psi}}} = 23.2 \text{ in.}$$

This would correspond to a 32 in. (4 course) high lintel. Since the top course would probably be grouted for prescriptive seismic reinforcement, grout the entire height of the lintel.

The effective depth d is calculated as the beam depth, 48 in., minus 4 in., or 44 in.

Determine required flexural reinforcement:

The factored moment is determined as:

$$M_u = \frac{w_u l^2}{8} = \frac{(1,710 \text{ lb/ft})(16.67 \text{ ft})^2 (12 \text{ in./ft})}{8} = 713,000 \text{ lb} - \text{in.}$$

The required area of flexural reinforcement can be determined from:

$$a = d - \sqrt{d^2 - \frac{2M_u}{0.8\varphi f'_m b}} = 44 \text{ in.} - \sqrt{(44 \text{ in.})^2 - \frac{2(713,000 \text{ lb-in.})}{0.8(0.9)(2,000 \text{ psi})(7.63 \text{ in.})}} = 1.50 \text{ in.}$$

$$A_{s,reqd} = \frac{0.8 f'_m ba}{f_y} = \frac{0.8(2,000 \text{ psi})(7.63 \text{ in.})(1.50 \text{ in.})}{60,000 \text{ psi}} = 0.305 \text{ in.}^2$$

Use 1-No. 5 bar (A_s = 0.31in.^{2}).

Check minimum reinforcement:

The modulus of rupture for fully grouted units, stress parallel to the bed joint, and Type S Portland cement/lime mortar is 267 psi. The cracking moment is:

$$M_{cr} = f_r S_n = f_r \frac{bh^2}{6} = 267 \text{ psi} \frac{7.63 \text{ in.}(48 \text{ in.})^2}{6} = 781,800 \text{ lb-in.}$$

1.3 times the cracking moment is 1.3(781,000 lb-in) = 1,016,000 lb-in. The nominal strength with 1-No. 5 is:

$$M_n = A_s f_y \left(d - \frac{1}{2} \frac{A_s f_y}{0.8 f'_m b} \right) = (0.31 \text{ in.}^2)(60,000 \text{ psi}) \left(44 \text{ in.} - \frac{1}{2} \frac{(0.31 \text{ in.}^2)(60,000 \text{ psi})}{0.8(2,000 \text{ psi})(7.63 \text{ in.})} \right)$$

$$= 804,000 \text{ lb-in.}$$

Thus, 1-No. 5 does not meet the minimum reinforcement requirements. Using 2-No. 4 bars (A_s=0.40in.^{2}) results in a nominal moment of 1,038,000 lb-in. and is sufficient.

Check maximum reinforcement:

Beams are required to be tension controlled. For 2-No. 4 bars, the reinforcement ratio is:

$$\rho = \frac{A_s}{bd} = \frac{2(0.20 \text{ in.}^2)}{7.63 \text{ in.}(44 \text{ in.})} = 0.00092$$

This is much less than the maximum reinforcement ratio of 0.00711, so the design is OK.

Check lateral support of lintel:

TMS 402 Section 5.3.1.3 requires lateral support of the compression face of the beam at the smaller of:

$$32b = 32(7.63 \text{ in.}) = 244 \text{ in.} = 20.3 \text{ ft}$$

$$120b^2 / d = 120(7.63 \text{ in.})^2 / 44 \text{ in.} = 159 \text{ in.} = 13.2 \text{ ft}$$

The roof framing provides lateral support for the compression face. If the top of the beam were a parapet, then lateral support would be needed at the midspan and ends of the lintel.

Check deflections:

TMS 402 Section 5.2.1.4 only requires the deflection to be checked if the lintel supports unreinforced masonry. The limiting deflection is ℓ /600. TMS 402 Section 5.3.1.6.1 states that deflections do not need to be checked if $\ell/d \leq 8$. For our lintel, ℓ/d = (16.67 ft × 12 in./ft) / 44 in. = 4.5 so deflections do not need to be checked even if the lintel is supporting unreinforced masonry, such as veneer.

Final Design: 2 – No. 4 bars

Strength design resulted in slightly less reinforcement being required than allowable stress design. This is due to most of the load being from dead load and the benefit of the 1.2 load factor on dead loads.

Rework problem with higher loads:

The lintel design is reworked with a uniformly distributed load of 2,000 lb/ft dead load and 700 lb/ft live load. This example will illustrate the use of multiple layers of flexural reinforcement to carry a large moment, and the use of shear reinforcement.

Determine required flexural reinforcement:

Distributed load: $w_u = 1.2\big(2000\text{ lb/ft} + 4\text{ ft}\big(81\text{ psf}\big)\big) + 1.6\big(700\text{ lb/ft}\big) = 3{,}910\text{ lb/ft}$

Applied moment: $M_u = \dfrac{w_u l^2}{8} = \dfrac{\big(3{,}910\text{ lb/ft}\big)\big(16.67\text{ ft}\big)^2\big(12\text{ in./ft}\big)}{8} = 1{,}630{,}000\text{ lb}-\text{in.}$

Assume two layers of reinforcement, which will cause d to be 40 in. The required area of flexural reinforcement can be determined from:

$$a = d - \sqrt{d^2 - \frac{2M_u}{0.8\varphi f'_m b}} = 40.0\text{ in.} - \sqrt{\big(40.0\text{ in.}\big)^2 - \frac{2\big(1{,}630{,}000\text{ lb}-\text{in.}\big)}{0.8\big(0.9\big)(2{,}000\text{ psi})(7.63\text{ in.})}} = 3.90\text{ in.}$$

$$A_{s,reqd} = \frac{0.8 f'_m ba}{f_y} = \frac{0.8\big(2{,}000\text{psi}\big)\big(7.63\text{ in.}\big)\big(3.90\text{ in.}\big)}{60{,}000\text{ psi}} = 0.793\text{ in.}^2$$

Use 4-No. 4 bars (A_s = 0.80in.^{2}), with 2 bars in each of the bottom two courses. ϕM_n = 1,640,000 lb-in.

Based on the previous example, the minimum reinforcement requirements are met by inspection.
The reinforcement ratio is 0.80in.^{2}/((7.63in.)(40in.)) = 0.00262, which is less than maximum reinforcement ratio of 0.00711.

Final Design: 4 – No. 4 bars, 2 each in bottom two courses

Determine required shear reinforcement:

TMS 402 Section 5.3.1.5 allows the lintel to be designed for the shear at d/2 from the face of support. The design shear force can be determined as follows, where ℓ_n is the clear span.

Shear: $V_u = w_u\left(\frac{\ell_n}{2} - \frac{d}{2}\right) = 3{,}910\frac{\text{lb}}{\text{ft}}\left(\frac{16\text{ ft}}{2} - \frac{40\text{ in.}(1\text{ ft}/12\text{in.})}{2}\right) = 24{,}760\text{ lb}$

Shear area: $A_{nv} = bd = 7.63\text{ in.}(40\text{ in.}) = 305\text{ in.}^2$

Nominal shear strength due to masonry:

$$V_{nm} = \left[\left(4 - 1.75\left(\frac{M_u}{V_u d_v}\right)\right)\right]A_{nv}\sqrt{f'_m} = \left[\left(4 - 1.75(1.0)\right)\right]\left(305\text{ in.}^2\right)\sqrt{2{,}000\text{ psi}} = 30{,}700\text{ lb}$$

Required nominal shear strength due to reinforcement:

$$\frac{V_u}{\phi} = \left(V_{nm} + V_{ns}\right)\gamma_g \quad \Rightarrow \quad V_{ns,reqd} = \frac{V_u}{\phi\gamma_g} - V_{nm} = \frac{24{,}760\text{ lb}}{0.8(1.0)} - 30{,}700\text{ lb} = 250\text{ lb}$$

Deformed wire is available in a variety of sizes ranging from D4 (0.04in.^{2}) to D31 (0.31in.^{2}) and could be used for the shear reinforcement. For this example, No. 3 bars will be used.

Determine spacing:

$$V_{ns} = 0.5\left(\frac{A_v}{s}\right)f_y d_v \quad \Rightarrow \quad s = \frac{0.5A_v f_y d_v}{V_{ns,reqd}} = \frac{0.5\left(0.11\text{ in.}^2\right)(60{,}000\text{ psi})(48\text{ in.})}{250\text{ lb}} = 634\text{ in.}$$

Due to the small required shear strength of the reinforcement, it could have been anticipated that the maximum spacing of stirrups would control.

TMS 402 Section 9.3.3.2.3 provides beam detailing requirements for shear reinforcement:

- The shear reinforcement shall be a single bar with a 180° hook at each end.
- The shear reinforcement shall be hooked around the longitudinal reinforcement.
- The minimum transverse area of shall be 0.0007*b*. This can be written as $A_v/s \geq 0.0007b$, or 0.11in.^{2}/8in.=0.0138in. ≥ 0.007(7.63in.) = 0.0053in., or OK.
- The first bar shall be located not more than one-fourth the beam depth, d_v, from the end of the beam. $0.25d_v$ = 0.25(48in.)=12in.
- The maximum spacing shall not exceed one half the depth of the beam, d_v, nor 48 inches. s_{max} = min{0.5(48in.),48in.}=24in.

To avoid the use of stirrups, a higher f'_m will be used. Setting $V_u = \phi V_{nm}$:

$$V_u = \phi V_{nm} = 0.8\left[\left(4 - 1.75\left(\frac{M_u}{V_u d_v}\right)\right)\right]A_{nv}\sqrt{f'_m}$$

$$24{,}760\text{ lb} = 0.8\left[\left(4 - 1.75(1.0)\right)\right]\left(305\text{ in.}^2\right)\sqrt{f'_m}$$

Solving, a f'_m of 2,035 psi would be required to avoid shear reinforcement. This would require a unit strength of 2,090 psi if the unit strength method, Table 2 in TMS 602 was used. Almost all block have a higher unit strength than this, with most block being at least 2,600 psi. Designers are encouraged to verify specified strengths with local producers.

Example 17.2-5: Retaining Wall

A 6 ft high cantilever wall is to be constructed with 8 in. concrete masonry units and Type S PCL mortar. The wall is to be fully grouted. The backfill has an equivalent fluid density of 31 pounds per cubic foot. The reinforcement is to be placed such that there is a 2 in. cover (1¼ in. for face shell, ¼ in. for taper of face shell, ½ in. for coarse grout).

Loads:

Pressure at base of wall: $p = \gamma h = 31\dfrac{\text{lb}}{\text{ft}^3}(6\text{ ft}) = 186\dfrac{\text{lb}}{\text{ft}^2}$

Total horizontal force: $H = \dfrac{1}{2}ph = \dfrac{1}{2}\left(186\dfrac{\text{lb}}{\text{ft}^2}\right)(6\text{ ft}) = 558\dfrac{\text{lb}}{\text{ft}}$

First-order Moment: $M = \dfrac{1}{3}h(H) = \dfrac{1}{3}(6\text{ft})\left(558\dfrac{\text{lb}}{\text{ft}}\right) = 1{,}116\dfrac{\text{lb-ft}}{\text{ft}}$

First-order factored moment: $M_u = 1.6M = 1.6\left(1{,}116\dfrac{\text{lb-ft}}{\text{ft}}\right) = 1{,}786\dfrac{\text{lb-ft}}{\text{ft}} = 21{,}430\dfrac{\text{lb-in.}}{\text{ft}}$

Material Properties: f'_m = 2,000 psi (TMS 402 Table 2); $E_m = 900f'_m$ = 900(2,000psi) = 1,800 ksi

The properties of the GFRP bars were obtained from the manufacturer. Use C_E = 0.8 for interior.

Bar size	f_{fu} (ksi)	f_{fd} (ksi)	E_f (ksi)	$\varepsilon_{fd} = f_{fd} / E_f$
No. 4	130	104	8,000	0.0130
No. 5	120	96	8,000	0.0120

Estimate Reinforcement:

Assume No. 5 bars:

$$d = t_{sp} - 2\text{ in.} - \frac{d_b}{2} = 7.625\text{ in.} - 2\text{ in.} - \frac{0.625\text{ in.}}{2} = 5.312\text{ in.}$$

Assume the strength is controlled by the masonry. Determine a.

$$a = d - \sqrt{d^2 - \frac{2M_u}{0.8\phi f'_m b}} = 5.312\text{ in.} - \sqrt{(5.312\text{ in.})^2 - \frac{2\left(21{,}430\dfrac{\text{lb}-\text{in.}}{\text{ft}}\right)}{0.8(0.75)(2{,}000\text{ psi})\left(12\dfrac{\text{in.}}{\text{ft}}\right)}} = 0.288\text{ in.}$$

Determine $c_b = \left(\dfrac{\varepsilon_{mu}}{\varepsilon_{mu} + \varepsilon_{fd}}\right)d = \left(\dfrac{0.0025}{0.0025 + 0.012}\right)5.312\text{ in.} = 0.916\text{ in.}$

If $c = a/0.8 > c_b$ then the strength is controlled by the masonry.

$$c = \frac{a}{0.8} = \frac{0.288\text{ in.}}{0.8} = 0.360\text{ in.}$$

Since $c < c_b$, the strength is controlled by the GFRP reinforcement. Try a No. 5 bar.

$$A_{f,reqd} = \frac{M_u}{\phi f_{fd}\left(d - \frac{0.8c_b}{2}\right)} = \frac{1{,}786\frac{\text{lb-ft}}{\text{ft}}\left(12\frac{\text{in.}}{\text{ft}}\right)}{0.55(96{,}000\text{ psi})\left(5.312\text{ in.} - \frac{0.8(0.916\text{ in.})}{2}\right)} = 0.082\frac{\text{in.}^2}{\text{ft}}$$

Try No. 5 at 40 in. $A_f = 0.093\ \text{in.}^2/\text{ft}$.

Design Strength:

Once a trial reinforcement is calculated, the design strength needs to be obtained. Since a greater amount of reinforcement is being used than estimated, it is necessary to verify that the strength is still being controlled by the GFRP reinforcement.

$$\rho_f = \frac{A_f}{bd} = \frac{0.093\text{ in.}^2/\text{ft}}{(12\text{ in./ft})5.312\text{ in.}} = 0.00146$$

$$\rho_{fb} = \frac{0.64f'_m}{f_{fd}}\left(\frac{\varepsilon_{mu}}{\varepsilon_{mu}+\varepsilon_{fd}}\right) = \frac{0.64(2{,}000\text{ psi})}{96{,}000\text{ psi}}\left(\frac{0.0025}{0.0025+0.012}\right) = 0.00230$$

Since $\rho_f \le \rho_{fb}$ the strength is controlled by the GFRP reinforcement.

$$c_b = \left(\frac{\varepsilon_{mu}}{\varepsilon_{mu}+\varepsilon_{fd}}\right)d = \left(\frac{0.0025}{0.0025+0.012}\right)5.312\text{ in.} = 0.916\text{ in.}$$

$$M_n = A_f f_{fd}\left(d - \frac{0.8c_b}{2}\right) = 0.093\frac{\text{in.}^2}{\text{ft}}(96\text{ ksi})\left(5.312\text{ in.} - \frac{0.8(0.916\text{ in.})}{2}\right) = 44.14\frac{\text{k-in.}}{\text{ft}}$$

$\phi = 0.55$ since tension-controlled

$$\phi M_n = 0.55\left(44.14\frac{\text{k-in.}}{\text{ft}}\right) = 24.28\frac{\text{k-in.}}{\text{ft}}$$

Creep rupture:

Since the soil pressure is a sustained load, check creep rupture.

The sustained moment, $M_{s,s}$, is the allowable stress moment or 1,116 lb-ft.

Modular ratio: $$n = \frac{E_f}{E_m} = \frac{8{,}000{,}000\text{ psi}}{900(2{,}000\text{ psi})} = \frac{8{,}000{,}000\text{ psi}}{1{,}800{,}000\text{ psi}} = 4.44$$

Find k: $n\rho_f = 4.44(0.0146) = 0.00648$.

$$k = \sqrt{(n\rho_f)^2 + 2n\rho_f} - n\rho_f = \sqrt{0.00648^2 + 2(0.00648)} - 0.00648 = 0.1076$$

Stress in reinforcement:

$$f_{f,s} = \frac{M_{s,s}}{A_f(1-k/3)d} = \frac{1{,}116\text{ lb-ft}\left(12\frac{\text{in.}}{\text{ft}}\right)}{0.093\frac{\text{in.}^2}{\text{ft}}\ (1-0.1384/3)5.312\text{ in.}} = 28{,}100\text{ psi}$$

The allowable stress under sustained loads is $0.3f_{fd}$ = 0.3(96,000 psi) = 28,800 psi which is greater than 28,100 psi; OK.

Second-order effects:
The deflection at the top of a cantilever, δ_u, for a triangular load is as follows, where w_u is the distributed load at the bottom of the wall:

$$\delta_u = \frac{1}{30}\frac{w_u h^4}{EI}$$

The first-order moment, $M_{u,0}$, at the base of the cantilever is:

$$M_{u,0} = \frac{1}{6} w_u h^2$$

Substituting this into the deflection equation:

$$\delta_u = \frac{1}{5}\frac{M_u h^2}{EI}$$

Two equations can now be written that are equivalent to TMS 402 Equations 9-21 and 9-24.

$$M_u = \frac{1}{6} w_u h^2 + \frac{P_u}{2}\delta_u$$

$$\delta_u = \frac{1}{5}\frac{M_{cr} h^2}{E_m I_n} + \frac{1}{5}\frac{(M_u - M_{cr})h^2}{E_m I_{cr}}$$

where P_u is the factored wall weight (half assumed to be acting at the top of the wall).

Solving the simultaneous linear equations for M_u and δ_u, the factored moment can be obtained as:

$$M_u = \frac{\frac{1}{6} w_u h^2 + \frac{P_u}{2}\frac{1}{5}\frac{M_{cr} h^2}{E_m}\left(\frac{1}{I_n} - \frac{1}{I_{cr}}\right)}{1 - \frac{P_u}{2}\frac{1}{5}\frac{h^2}{E_m I_{cr}}}$$

Section Properties: from NCMA TEK 14-1B for 8 inch CMU and a full grouting.

A_n = 91.5 in.^{2}/ft; S_n = 116.3 in.^{3}/ft; I_n = 443.3 in.^{4}/ft

Wall Weight: 81 psf (ASCE-7 or NCMA TEK 13-B)

Wall weight, $D = 6 \text{ ft}(81 \text{ psf}) = 486 \text{ lb/ft}$

$$P_u = 0.9D = 0.9(486 \text{ lb/ft}) = 437 \text{ lb/ft}$$

$$P_u = 1.2D = 1.2(486 \text{ lb/ft}) = 583 \text{ lb/ft}$$

Cracking Moment:
Type S PCL, tension normal to the bed joint, TMS 402 Table 9.1.9.1

Fully Grouted: f_r = 163 psi

Cracking Moment: Use minimum axial load as this will result in the lowest cracking moment.

$$M_{cr} = \left(\frac{P_u}{A_n} + f_r\right) S_n = \left(\frac{437 \text{ lb/ft}}{91.5 \text{ in.}^2/\text{ft}} + 163 \text{ psi}\right) 116.3 \text{ in.}^3/\text{ft} = 19{,}510 \text{ lb-in./ft}$$

<u>Cracked Moment of Inertia:</u>
Use the value of c that was used for strength calculations and the modular ratio from above.

$$I_{cr} = nA_f\left(d-c\right)^2 + \frac{nP_u}{f_f}\left(\frac{t_{sp}}{2} - c\right)^2 + \frac{bc^3}{3} = 4.44\left(0.093 \text{ in.}^2/\text{ft}\right)\left(5.312 \text{ in.} - 0.916 \text{ in.}\right)^2$$

$$+ \frac{4.44(583 \text{ lb/ft})}{96{,}000 \text{ psi}}\left(\frac{7.625 \text{ in.}}{2} - 0.916 \text{ in.}\right)^2 + \frac{\left(12 \text{ in./ft}\right)\left(0.916 \text{ in.}\right)^3}{3} = 12.28 \text{ in.}^4/\text{ft}$$

<u>Factored Moment:</u>

$$M_u = \frac{\frac{1}{6}w_u h^2 + \frac{P_u}{2}\frac{1}{5}\frac{M_{cr}h^2}{E_m}\left(\frac{1}{I_n} - \frac{1}{I_{cr}}\right)}{1 - \frac{P_u}{2}\frac{1}{5}\frac{h^2}{E_m I_{cr}}}$$

$$= \frac{21{,}430\frac{\text{lb-in.}}{\text{ft}} + \frac{\left(583\frac{\text{lb}}{\text{ft}}\right)\left(19{,}510\frac{\text{lb-in.}}{\text{ft}}\right)\left(6 \text{ ft}\right)^2\left(12\frac{\text{in.}}{\text{ft}}\right)^2}{2\left(5\right)\left(1{,}800{,}000 \text{ psi}\right)}\left(\frac{1}{443.3\frac{\text{in.}^4}{\text{ft}}} - \frac{1}{12.28\frac{\text{in.}^4}{\text{ft}}}\right)}{1 - \frac{\left(583\frac{\text{lb}}{\text{ft}}\right)\left(6 \text{ ft}\right)^2\left(12\frac{\text{in.}}{\text{ft}}\right)^2}{2\left(1{,}800{,}000 \text{ psi}\right)\left(12.28\frac{\text{in.}^4}{\text{ft}}\right)}}$$

$$= 22{,}710\frac{\text{lb-in.}}{\text{ft}} = 22.71\frac{\text{k-in.}}{\text{ft}}$$

The factored moment, 22.71 kip-in./ft, is 0.94 times the design moment, ϕM_n = 24.28 kip-in./ft. The design is OK.

The allowable stress moment, 13.39 kip-in./ft, is less than the cracking moment of 19.51 kip-in./ft. Therefore, deflections are OK.